Equivalence of Light-Front and Covariant Approaches in Meson-Baryon Interactions

C.Ji, W.Melnitchouk, A.Thomas, PRD80, 054018(09)

B.Bakker, M.DeWitt, C.Ji, Y.Mishchenko, PRD72, 076005(05)

College of William and Mary, Williamsburg, Virginia,
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Motivation

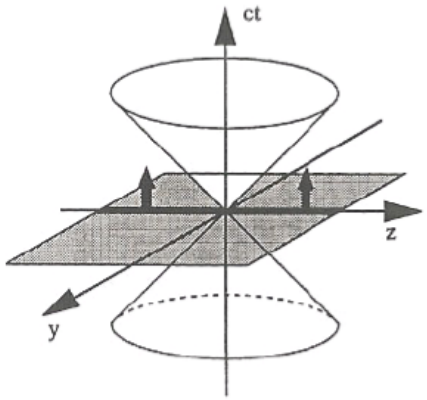
- Guidance from Approximate Chiral Symmetry of QCD
  - -Extrapolation of Lattice Results for Small Pion Mass
  - -Pion Cloud Contribution in Chiral Perturbation Theory (X_{PT})
  - -Leading Non-Analytic (LNA) Behavior of Goldstone Boson Loops:
    Gell-Mann – Oaks – Renner Relation: $m_\pi^2 f_\pi^2 = 2m_q < \bar{q}q >$
- Great Interest in Generalized Parton Distributions (GPDs)
  - -Most Natural in Light-Front Dynamics (LFD)
  - -Infinite Momentum vs. Rest Frame as well as
    Pseudovector(PV) vs. Pseudoscalar(PS) Coupling in
    Equal-Time Calculations and Subtle Discrepancy between Results
    A.Thomas, W.Melnitchouk, F.Steffens, PRL85, 2892(00);
    J.-W. Chen, X.Ji, PRL87, 152002(01);
    M.Burkardt,private communication, etc.
- Discussion on Chiral Symmetry in LFD
  - -Distinguished Features: Vacuum, Symmetry
  - -Care of Treacherousness: Zero-Modes, End-Point Singularities, …

Outline

- Distinguished Features in LFD
  - Energy-Momentum Dispersion Relation
  - Common Belief of Equivalence
  - Treacherous Amplitudes
- Nucleon Self-Energy in χ_{PT} (LNA Behaviors)
  - Manifestly Covariant Approach
  - Equal-Time Approach

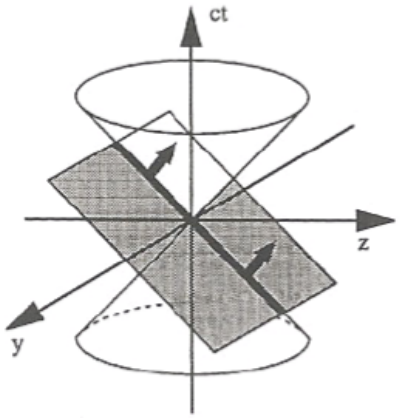
    Rest Frame vs. Infinite Momentum Frame
  - LFD Approach
- Conclusion and Outlook

Distinguished Features in LFD



The instant form

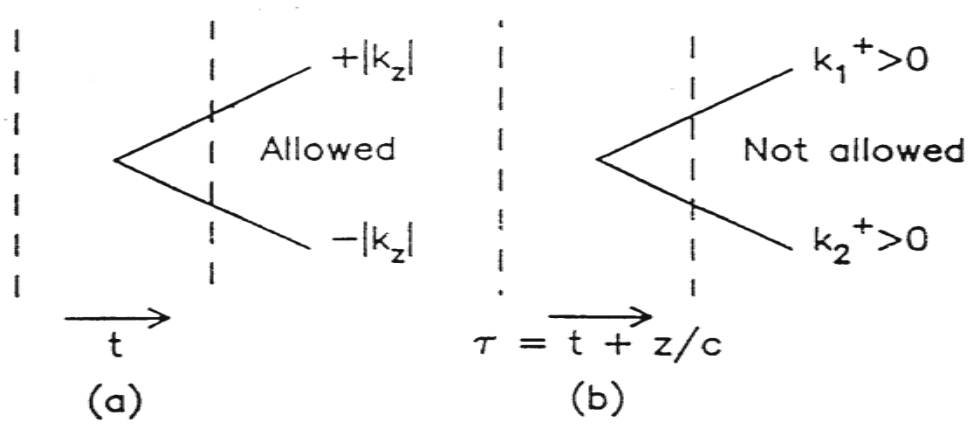
Equal t		*Equal τ*
p^0	$\leftrightarrow$	$p^- = p^0 - p^3$
(p^1, p^2)	$\leftrightarrow$	$\vec{p}_\perp$
p^3	$\leftrightarrow$	$p^+ = p^0 + p^3$



The front form

Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2} \qquad\qquad p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



Common Belief of Equivalence

Manifestly Covariant Formulation

$\int dk^0$ $\int dk^-$

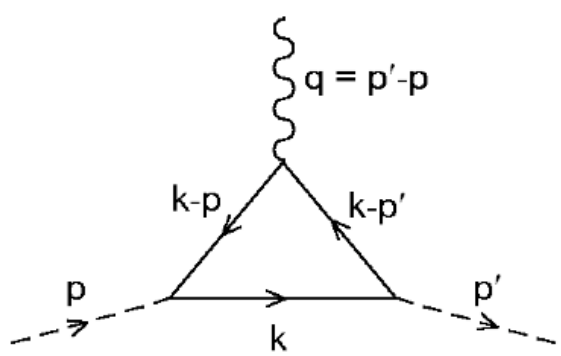
Equal t Formulation — Equal τ = t + z/c Formulation

$\sum$ (Time Ordered Amps)

However, the proof of equivalence is treacherous.

Example: Arc-contribution in physical form factor

B.Bakker, M.DeWitt, C.Ji, Y.Mishchenko, PRD72, 076005(05)

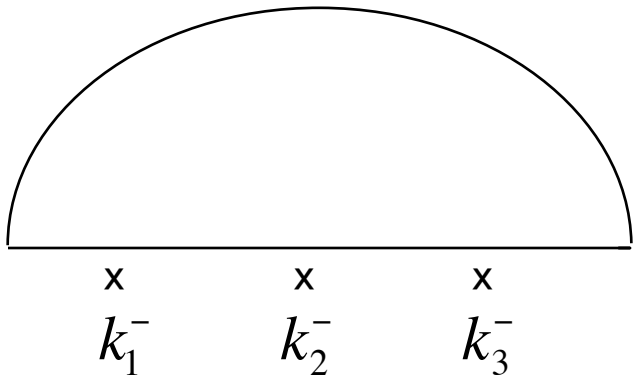


$$< p' \mid J^\mu \mid p >= i(p^\mu + p'^\mu)F(q^2)$$

$$\propto \int d^2k \frac{k^\mu k^2 + \cdots}{D_1 D_2 D_3}$$

$$< p' | J^+ | p >= i(p^+ + p'^+)F(q^2) \propto \int dk^+ dk^- \frac{k^+ k^+ k^- + \cdots}{D_1 D_2 D_3}$$

$$< p' | J^- | p >= i(p^- + p'^-)F(q^2) \propto \int dk^+ dk^- \frac{k^- k^+ k^- + \cdots}{D_1 D_2 D_3}$$



$$\oint_{contour} dk^- = \int_{-\infty}^{+\infty} dk^- + \int_{arc} dk^- = 0$$

$$\int_{-\infty}^{+\infty} dk^- = -\int_{arc} dk^-$$

$$\int_{-\infty}^{\infty} dk^- \frac{(k^-)^2}{(k^- - k_1^-)(k^- - k_2^-)(k^- - k_3^-)} = -i \int_{arc} d\theta = -i\pi$$

With the arc contribution, we confirmed the equivalence between covariant and light-front approach.

Treacherous Amplitudes in Nucleon Self-Energy

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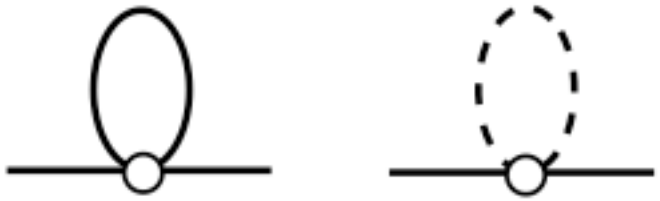


$$I = \int d^2k \frac{1}{k^2 - m^2 + i\varepsilon}$$

Manifestly Covariant Calculation

$$I = -i\pi\left(\frac{2-n}{2} - \log\pi - \gamma + O(2-n) - \log\frac{m^2}{\mu^2}\right)_{n\to 2}$$

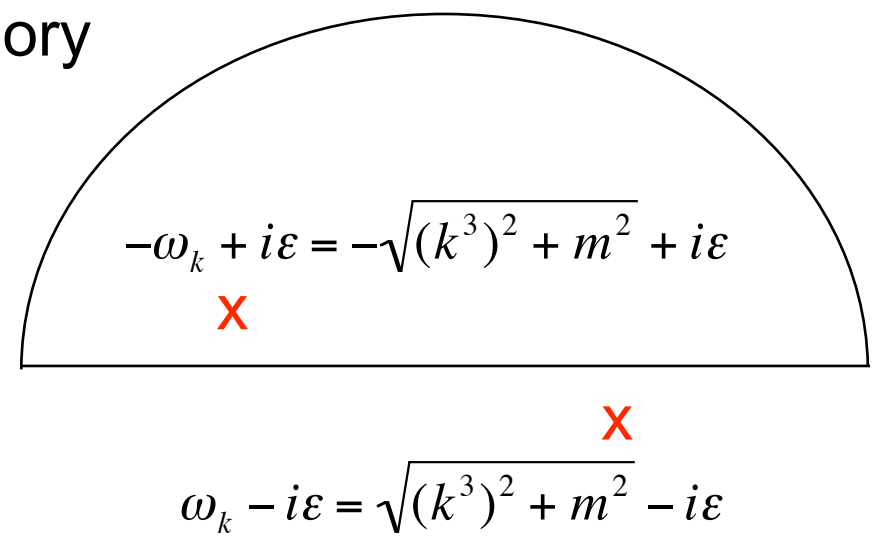
$$I_{LNA} = i\pi \log m^2$$

Time-Ordered Perturbation Theory

$$I = \int dk^3 dk^0 \frac{1}{(k^0 + \omega_k - i\varepsilon)(k^0 - \omega_k + i\varepsilon)}$$

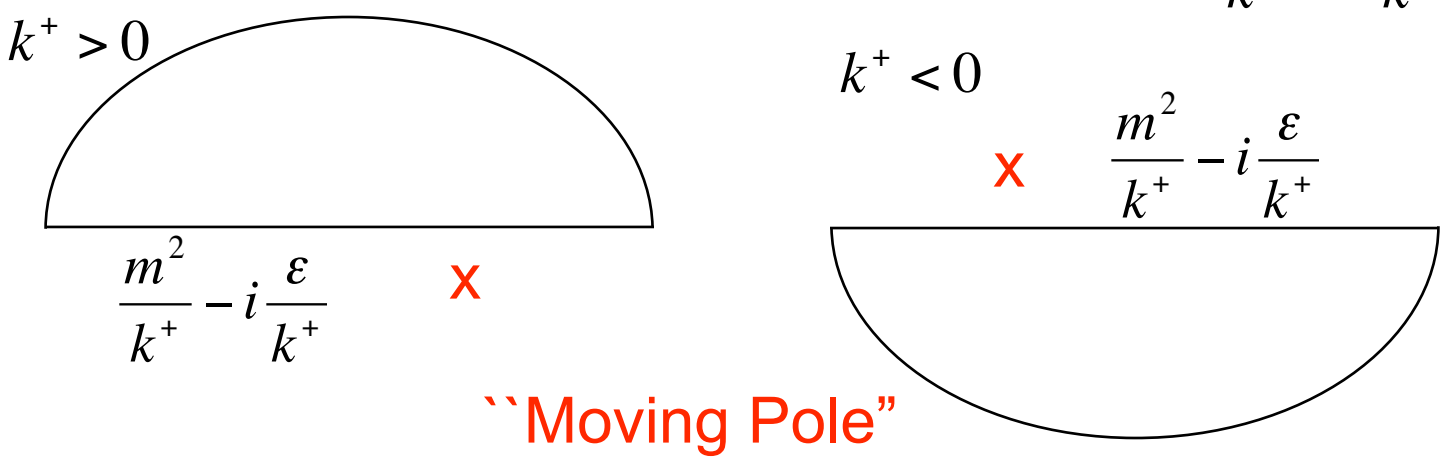
$$= -2i\pi \int_0^{\Lambda\to\infty} dk^3 \frac{1}{\sqrt{(k^3)^2 + m^2}}$$

$$= -2i\pi \log\left(\frac{2\Lambda}{m}\right)_{\Lambda\to\infty}$$



LFD

$$I = \frac{1}{2}\int dk^+ dk^- \frac{1}{k^+k^- - m^2 + i\varepsilon} = \frac{1}{2}\int \frac{dk^+}{k^+}\int dk^- \frac{1}{k^- - \frac{m^2}{k^+} + i\frac{\varepsilon}{k^+}}$$



LF Polar Coordinate $\quad k^+ = r\cos\phi \quad k^- = r\sin\phi$

$$I = \int_0^\infty dr\, r \int_0^{2\pi} d\phi \frac{1}{r^2 \sin\phi\cos\phi - m^2 + i\varepsilon} = -i\pi\left(\log\frac{R^2 e^{-i\pi/2}}{2m^2} + O(1/R^4)\right)_{R\to\infty}$$

$$I_{LNA}^{LF} = i\pi \log m^2$$

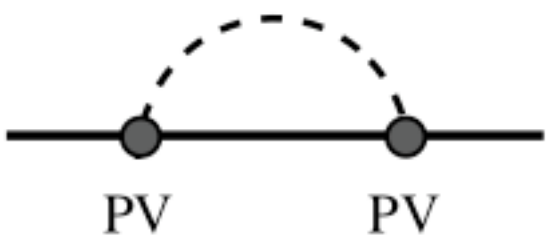
πN Interaction with PS vs. PV Coupling

$$L_{PS} = -g_{\pi NN}(\overline{\psi}_N i\gamma_5 \vec{\tau}\psi_N)\cdot\vec{\phi}_\pi \quad ; \quad L_{PV} = \frac{f_{\pi NN}}{m_\pi}(\overline{\psi}_N \gamma^\mu \gamma_5 \vec{\tau}\psi_N)\cdot\partial_\mu\vec{\phi}_\pi$$

$$\frac{g_{\pi NN}}{2M} = \frac{f_{\pi NN}}{m_\pi} \quad , \quad \frac{g_A}{f_\pi} = \frac{g_{\pi NN}}{M}$$

(Goldberger-Treiman Relation)

Coupling Constants; $g_A \approx 1.267,\ f_\pi \approx 93\,MeV,\ \frac{g^2_{\pi NN}}{4\pi} \approx 14.3\,,\ \frac{f^2_{\pi NN}}{4\pi} \approx 0.08$



$$\Sigma^{PV} = \frac{1}{2}\sum_s \overline{u}(p,s)\,\hat{\Sigma}^{PV}\,u(p,s)$$

$$\hat{\Sigma}^{PV} = -i\left(\frac{2g_A}{f_\pi}\right)^2 \vec{\tau}\cdot\vec{\tau}\int \frac{d^4k}{(2\pi)^4}\frac{\not{k}\gamma_5(\not{p}-\not{k}+M)\gamma_5\not{k}}{D_\pi D_N}$$

$$D_\pi = k^2 - m_\pi^2 + i\varepsilon \qquad D_N = (p-k)^2 - M^2 + i\varepsilon$$

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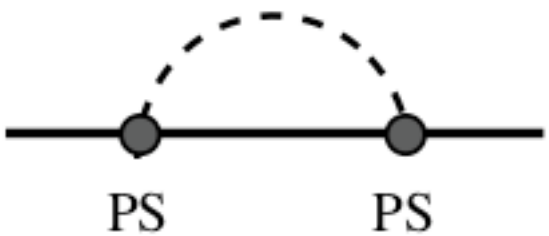

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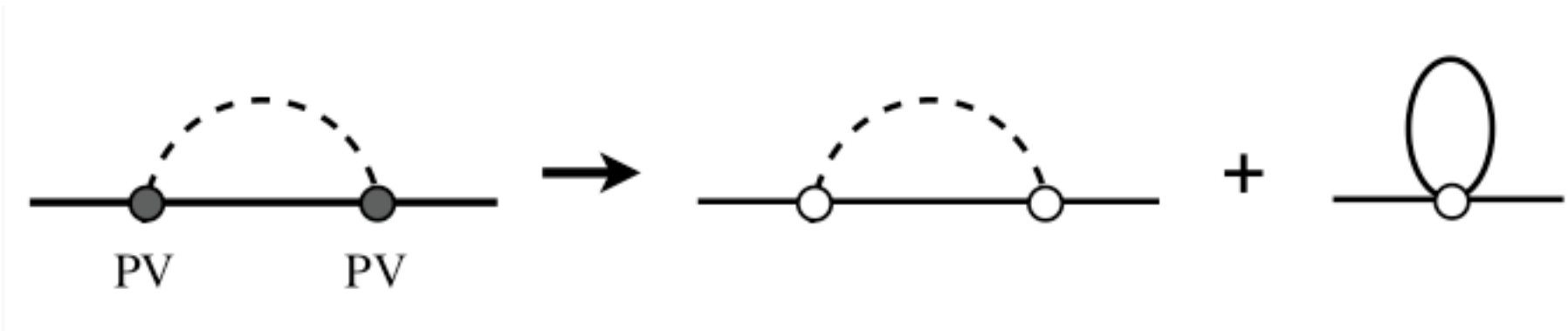
$$D_\pi = k^2 - m_\pi^2 + i\varepsilon \qquad D_N = (p-k)^2 - M^2 + i\varepsilon$$

Nucleon Self-Energy

$$\hat{\Sigma} = \Sigma_V \not{p} + \Sigma_S \Longrightarrow \Sigma = M\Sigma_V + \Sigma_S$$

Substitution Technique: $k^2 \to D_\pi + m_\pi^2 \; ; \; p \cdot k \to \frac{1}{2}(D_\pi - D_N + m_\pi^2)$



$$M\Sigma_V^{PV} = \Sigma_S^{PV} = \frac{1}{2}\Sigma^{PV} = -\frac{3ig_A^2}{4f_\pi^2}\int \frac{d^4k}{(2\pi)^4} M\left[\frac{1}{D_N} + \frac{m_\pi^2}{D_\pi D_N}\right]$$

Nucleon Self-Energy

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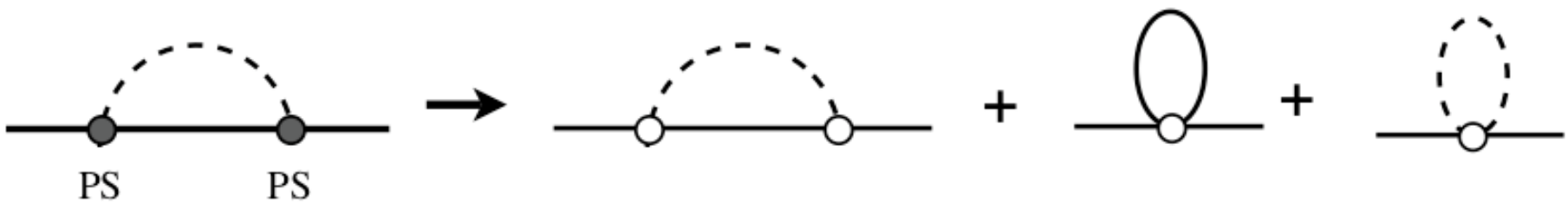


$$\Sigma^{PS} = -\frac{3ig_A^2 M}{2f_\pi^2}\int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{D_N} - \frac{1}{D_\pi} + \frac{m_\pi^2}{D_\pi D_N}\right]; \Sigma_V^{PS} = -3i\left(\frac{Mg_A}{f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{D_\pi D_N},$$

$$\Sigma_S^{PS} = -3i\left(\frac{Mg_A}{f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{2M}\left[\frac{1}{D_N} - \frac{1}{D_\pi} + \frac{m_\pi^2 - 2M^2}{D_\pi D_N}\right]$$

Manifestly Covariant Calculation

$$\int d^4k \frac{1}{D_\pi D_N} = \int_0^1 dx \int d^4k \frac{1}{\left[xD_\pi + (1-x)D_N\right]^2}$$

$$= \int_0^1 dx \int d^4k' \frac{1}{\left[k'^2 + D_{\text{cov}} + i\varepsilon\right]^2}$$

$$= i\int_0^1 dx \int d^4\kappa \frac{1}{\left[\kappa^2 - D_{\text{cov}} - i\varepsilon\right]^2}$$

$$= i\mu^{4-n}\int_0^1 dx \int d^n\kappa \frac{1}{\left[\kappa^2 - D_{\text{cov}} - i\varepsilon\right]^2}$$

$$k \to k' = k - (1-x)p$$

$$D_{\text{cov}} = -(1-x)^2 M^2 - x\, m_\pi^2$$

$$d^4k' \to i\, d^4\kappa\,,\; k'^2 \to -\kappa^2$$

$$\int d^n\kappa \frac{(\kappa^2)^\beta}{(\kappa^2 + a^2)^\alpha} = \frac{\pi^{\frac{n}{2}}}{(a^2)^{\alpha-\beta-\frac{n}{2}}} \frac{\Gamma(\beta + \frac{n}{2})\Gamma(\alpha - \beta - \frac{n}{2})}{\Gamma(\frac{n}{2})\Gamma(\alpha)}$$

$$\int d^4k \frac{1}{D_\pi D_N} = -i\pi^2\left[\gamma + \log\pi - \frac{2}{4-n} + \int_0^1 dx \log\frac{(1-x)^2 M^2 + x\, m_\pi^2}{\mu^2} + O(4-n)\right]_{n\to 4}$$

$$\int_0^1 dx \log\frac{(1-x)^2 M^2 + x\, m_\pi^2}{\mu^2} = \log\frac{M^2}{\mu^2} + \frac{m_\pi^2}{2M^2}\log\frac{m_\pi^2}{M^2} + \frac{m_\pi\sqrt{4M^2 - m_\pi^2}}{M^2}\left(\tan^{-1}\frac{m_\pi}{\sqrt{4M^2 - m_\pi^2}} - \tan^{-1}\frac{m_\pi^2 - 2M^2}{m_\pi\sqrt{4M^2 - m_\pi^2}}\right) - 2$$

LNA Behavior as $m_\pi \to 0$

$$\tan^{-1}\frac{m_\pi^2 - 2M^2}{m_\pi\sqrt{4M^2 - m_\pi^2}} = -\frac{\pi}{2} + m_\pi + \frac{m_\pi^3}{24M^3} + O(\frac{m_\pi^5}{M^5}),$$

$$\frac{m_\pi\sqrt{4M^2 - m_\pi^2}}{M^2} = \frac{2m_\pi}{M}\left(1 - \frac{m_\pi^2}{8M^2} + O(\frac{m_\pi^4}{M^4})\right), \cdots$$

$$\Sigma^{PV} = -\frac{3ig_A^2}{2f_\pi^2}\int\frac{d^4k}{(2\pi)^4}M\left[\frac{1}{D_N} + \frac{m_\pi^2}{D_\pi D_N}\right]$$

$$\Sigma_{LNA}^{PV} = -\frac{3g_A^2}{32\pi f_\pi^2}\left[m_\pi^3 + \frac{m_\pi^4}{2\pi M}\log m_\pi^2 + O(m_\pi^5)\right]$$

Contribution from a Δ intermediate state $\propto \dfrac{m_\pi^4}{M_\Delta - M}\log m_\pi^2$

D.Leinweber,A.Thomas,K.Tsushima,S.Wright,PRD61,074502(00)

LNA Behavior as $m_\pi \to 0$

$$\tan^{-1}\frac{m_\pi^2 - 2M^2}{m_\pi\sqrt{4M^2 - m_\pi^2}} = -\frac{\pi}{2} + m_\pi + \frac{m_\pi^3}{24M^3} + O(\frac{m_\pi^5}{M^5}),$$

$$\frac{m_\pi\sqrt{4M^2 - m_\pi^2}}{M^2} = \frac{2m_\pi}{M}\left(1 - \frac{m_\pi^2}{8M^2} + O(\frac{m_\pi^4}{M^4})\right), \cdots$$

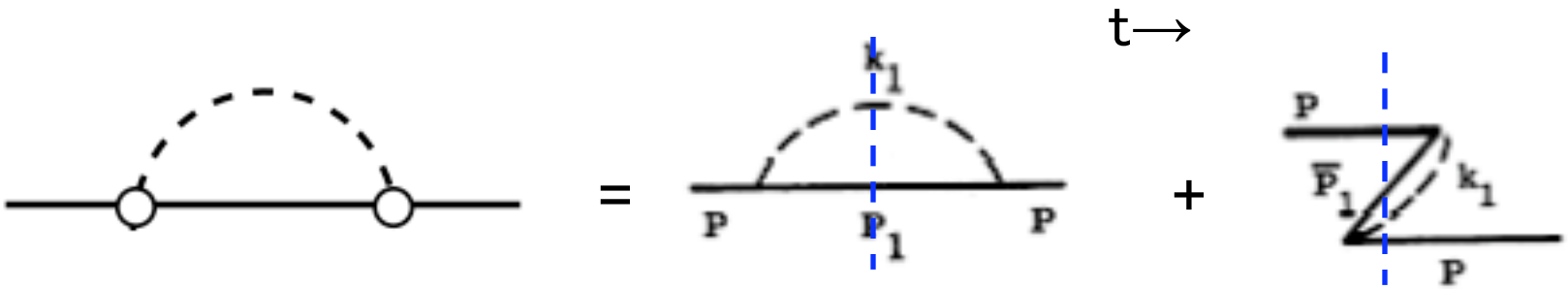
$$\Sigma^{PS} = -\frac{3ig_A^2 M}{2f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\left[\frac{1}{D_N} - \frac{1}{D_\pi} + \frac{m_\pi^2}{D_\pi D_N}\right]$$

$$\Sigma_{LNA}^{PS} = \frac{3g_A^2}{32\pi^2 f_\pi^2}\left[\frac{M}{\pi}m_\pi^2\log m_\pi^2 - m_\pi^3 + \frac{m_\pi^4}{2M}\log m_\pi^2 + O(m_\pi^5)\right]$$

$$M\Sigma_{V,LNA}^{PS} = \frac{3g_A^2 M}{32\pi^2 f_\pi^2}\left[2M^2 m_\pi + \frac{2M}{\pi}m_\pi^2\log m_\pi^2 - m_\pi^3 + \cdots\right]$$

$$\Sigma_{S,LNA}^{PS} = -\frac{3g_A^2}{32\pi^2 f_\pi^2}\left[2M^2 m_\pi + \frac{M}{\pi}m_\pi^2\log m_\pi^2 + \cdots\right]$$

Time-Ordered Perturbation Theory



$$\frac{1}{D_\pi D_N} = \frac{1}{(-2)(\omega_k - i\varepsilon)}\left(\frac{1}{k^0 - \omega_k + i\varepsilon} - \frac{1}{k^0 + \omega_k - i\varepsilon}\right)\frac{1}{2(E' - i\varepsilon)}\left(\frac{1}{k^0 - E + E' - i\varepsilon} - \frac{1}{k^0 - E - E' + i\varepsilon}\right)$$

$$\omega_k = \sqrt{\vec{k}^2 + m_\pi^2} \quad , \quad E = \sqrt{\vec{p}^2 + M^2} \quad , \quad E' = \sqrt{(\vec{p} - \vec{k})^2 + M^2}$$

$$\Sigma_{ET}^{(+-)} \quad \frac{\times \quad E - E' + i\varepsilon}{\omega_k - i\varepsilon \quad \times}$$

$$\Sigma_{ET}^{(-+)} \quad \frac{\times \quad -\omega_k + i\varepsilon}{E + E' - i\varepsilon \quad \times}$$

$$\Sigma_{ET}^{(++)} \quad \frac{\omega_k - i\varepsilon \qquad E + E' - i\varepsilon}{\times \qquad\qquad \times}$$

$$\Sigma_{ET}^{(--)} \quad \frac{\times \qquad\qquad \times}{-\omega_k + i\varepsilon \qquad E - E' + i\varepsilon}$$

Equal-Time Results

$$\Sigma_{ET}^{(+-)LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left[m_\pi^3 + \frac{3m_\pi^4}{4\pi M}\log m_\pi^2 + O(m_\pi^5)\right]$$

$$\Sigma_{ET}^{(-+)LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left[\quad - \frac{m_\pi^4}{4\pi M}\log m_\pi^2 + O(m_\pi^5)\right]$$

Without using the substitution technique, we get

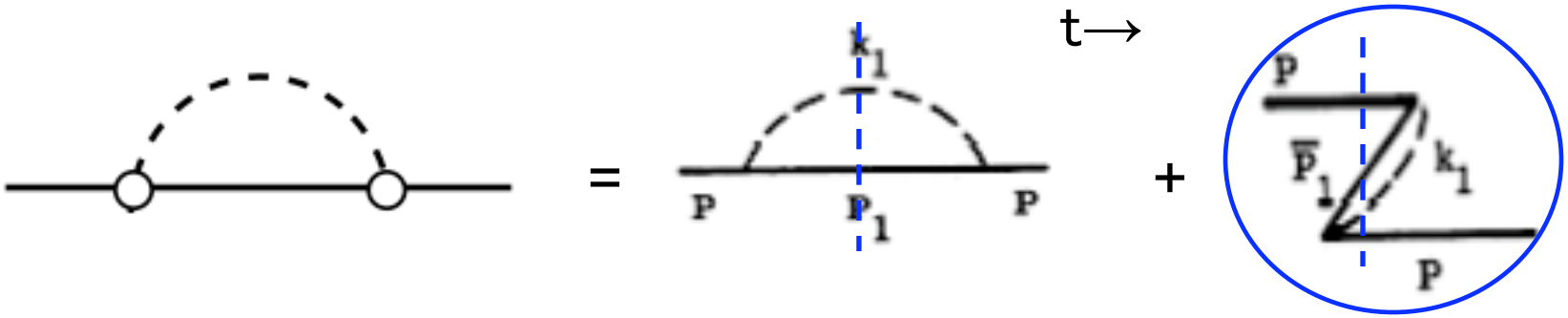
$$\Sigma_{ET}'^{(+-)LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left[\quad m_\pi^3 \quad + \frac{13m_\pi^4}{16\pi M}\log m_\pi^2 + O(m_\pi^5)\right]$$

$$\Sigma_{ET}'^{(-+)LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left[-\frac{1}{2\pi}Mm_\pi^2\log m_\pi^2 - \frac{5m_\pi^4}{16\pi M}\log m_\pi^2 + O(m_\pi^5)\right]$$

$$\Sigma_{ET}'^{(++)LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left[\quad \frac{1}{2\pi}Mm_\pi^2\log m_\pi^2 + \frac{m_\pi^4}{8\pi M}\log m_\pi^2 + O(m_\pi^5)\right]$$

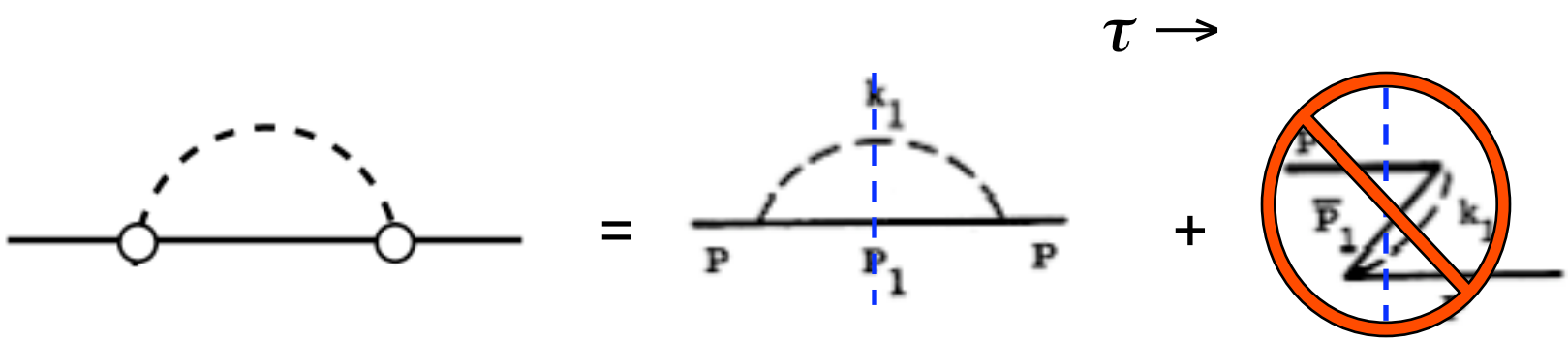
$$\Sigma_{ET}'^{(--)LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left[\quad - \frac{m_\pi^4}{8\pi M}\log m_\pi^2 + O(m_\pi^5)\right]$$

Infinite Momentum Frame



$$\vec{p} \to \infty \quad \Sigma^{(-+)}_{IMF} = O(1/\vec{p}^{\,2})$$

Light-Front Dynamics



$$\Sigma^{LNA}_{LF} = \Sigma^{(+-)LNA}_{IMF} = \Sigma^{(+-)LNA}_{ET} + \Sigma^{(-+)LNA}_{ET} = \Sigma^{LNA}_{COV}$$

Conclusion and Outlook

- Equivalence of the covariant, equal-time (in both the rest frame and in the IMF), and LF formalisms is highly nontrivial.

  -Arc contributions, moving poles, and end-point singularities.

- LNA behaviors of the vertex renormalization and the moments of the twist-two parton distribution functions are underway.

  -Form factors, PDFs and GPDs.

- Hope to pave the way for a consistent interpretation of the physics of the pion cloud at the partonic level.